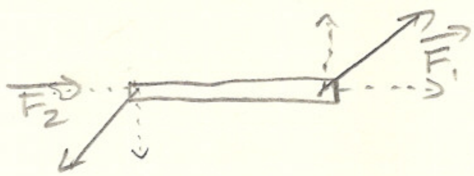
§ 6-3 EQUILIBRIUM IN TWO DIMENSIONS.

$$R = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} = 0$$
$$C = \Sigma M_z k = 0$$

we know

$$\Sigma F_z = 0 \quad \Sigma M_x = 0 \quad \Sigma M_y = 0$$

§ 6-3.1 TWO FORCE SYSTEMS. (TWO FORCE MEMBER).



$$\vec{F_1} = F_{1x}\vec{i} + F_{1y}\vec{j}$$
$$\vec{F_2} = F_{2x}\vec{i} + F_{2y}\vec{j}$$

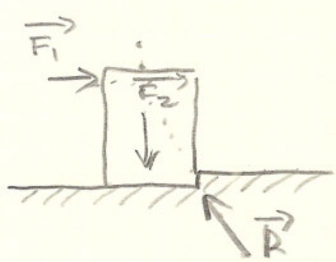
$$R_x = \Sigma F_x = F_{x_1} - F_{x_2} = 0$$

$$R_y = \Sigma F_y = F_{1y} - F_{2y} = 0$$

$$M_1 = -F_{2y} \; A.B = 0$$

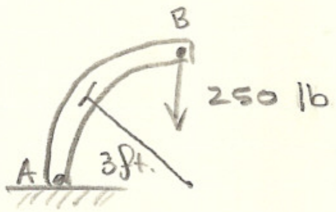
$$\therefore \; F_{y_1} = F_{y_2} = 0$$
$$F_{x1} = F_{x2}$$

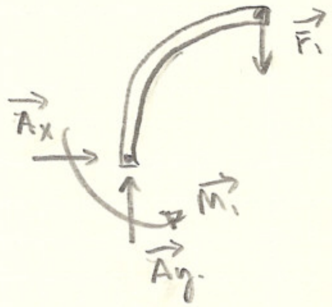
§ 6-3.2 THREE FORCE SYSTEM (THREE FORCE MEMBERS)



THE FORCES MUST BE CONCURRENT. THEY MUST PASS THROUGH THE SAME POINT.

EX.





$A_x = 0$

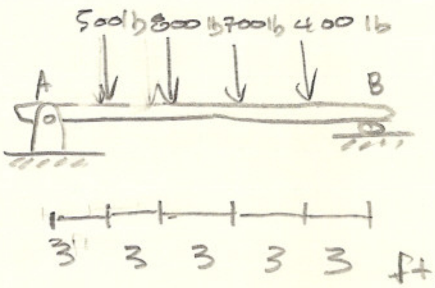
$R_y = A_y - F_1 = 0$

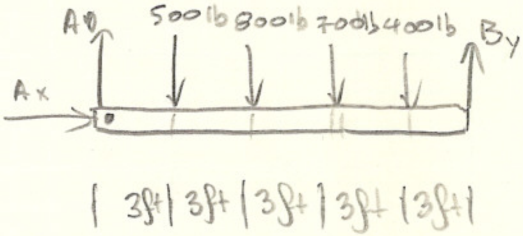
$A_y = 250 \text{ lb.}$

$\Sigma M = 0 = M_A - M_B = 0$

$M_A = 750 \text{ lb ft}$

EX.





$R_x = A_x = 0.$

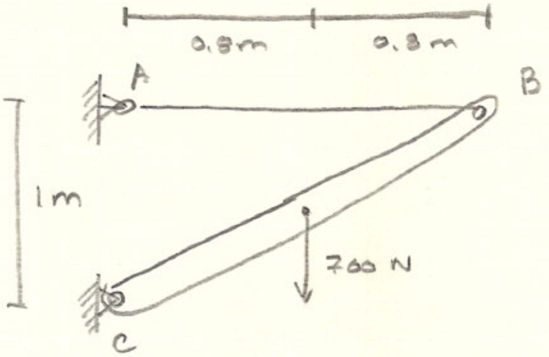
$R_y = A_y + B_y - 500 - 800 - 700 - 400$

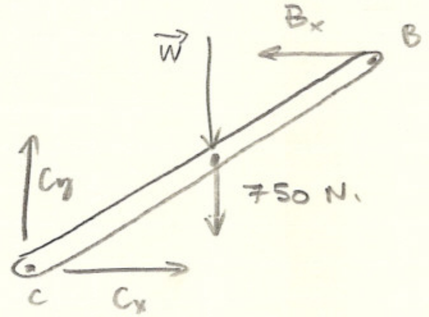
$M_A = (B_y \cdot 15) - (500 \cdot 3) - (800 \cdot 6)$
$- (700 \cdot 9) - (400 \cdot 12)$

$B_y = 1160 \text{ lb } \vec{j}$

$A_y = 1240 \text{ lb. } \vec{j}$

EX.





$$R_y = 0 = -W - 750\,N + C_y$$
$$= -490 - 750 + C_y$$
$$C_y = 1241\,N$$

$$\Sigma M_c = 0 = B_x(1m) - (1241)(450)(0.8m)$$
$$B_x = 992.4\,N$$

$$R_x = C_x - B_x$$

$$C_x = 992.4\,N$$

CHAPTER 7 TRUSSES, FRAMES AND MACHINES.

§ 7.1 SIMPLE TRUSSES.

A truss, a structure composed of slender members joined together at their end points and loaded only at their joints

Assumptions.

- TRUSS members are connected at their ends only

- Truss members are conected together by frictionless pins
- The weight of the members may be neglected.

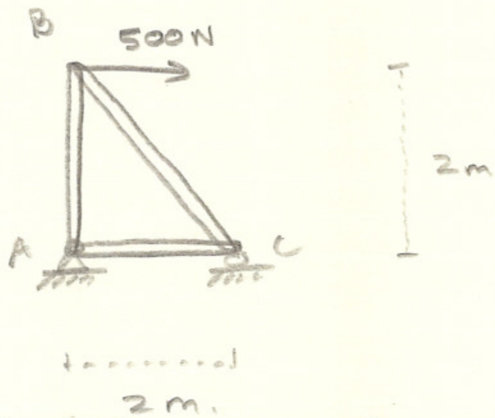
note that each truss member acts as a two force system.

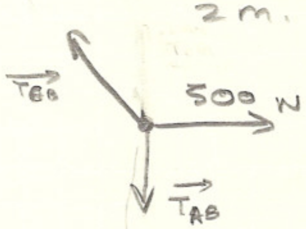
each pin must have 2 members attached.

§ 7.2 METHOD OF JOINTS.

- DRAW FBD FOR EACH JOINT.
- ANALYSIS STARTS A POINT HAVING A LEAST ONE KNOWN FORCE AND AT MOST 2 UNKNOWN FORCES.

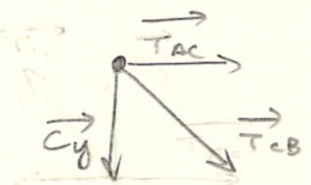
EX,



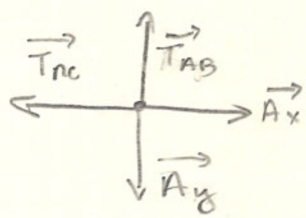


JOINT B.

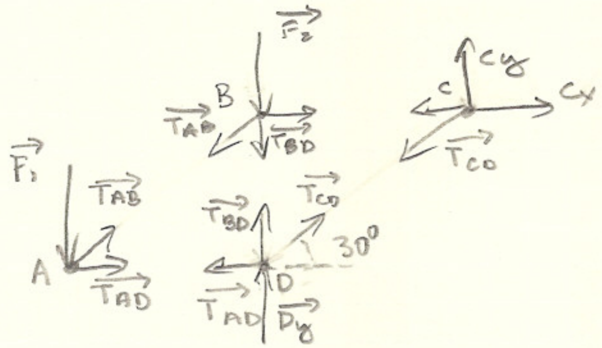
JOINT C



JOINT A.



EX.



Determine the force in each member. State weather each member is in Tension or compression.

Joint A.

$\vec{F} = -800\vec{j}$ N.

$R_x = 0 = -800j \cdot T_{AB} \sin 30° \vec{j}$

$T_{AB} = 1.60$ kN.

$R_y = T_{AB} \cos 30° + T_{AD}$

$T_{AD} = -1.39$ kN.

Joint B. $\vec{F_2} = -1000\vec{j}$ N.

$\vec{T_{AB}} = 1600(-\cos 30\vec{i} - \sin 30°\vec{j})$

$R_x = 0 = -1600 \cos 30° \vec{i} + T_{CB}\vec{i}$

$T_{CB} = 1385.6$ N

$R_y = -1000\vec{j} - 1600 \sin 30 j + T_{BD}$

$T_{BD} = -1800$ N.

Joint D.

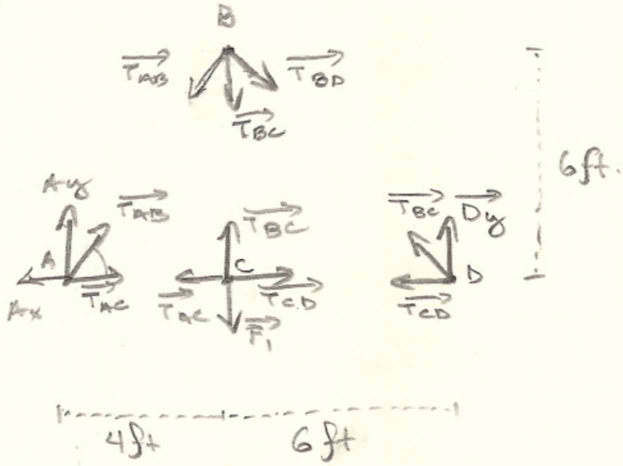
$R_x = -1385\vec{i} + T_{CD} \cos 30° \vec{i} = 0$

$T_{CD} = 1600$ N

$R_y = 0 = -1800\vec{j} + D_y\vec{j}$

$D_y = +1800\vec{j}$ N.

EX,



$F_1 = 1000$ lb.

$$\Sigma \vec{F_x} = 0$$

$$\Sigma \vec{F_y} = 0$$

$$\Sigma \vec{M_A} = 0$$

b/c there is nowheres with 2 unknowns and 1 known. We look at the moment. And we must look at the truss as one structure.

$$\Sigma \vec{M_A} = 0 = D_y(10) - 1000(4)$$

$\therefore D_y = 400$ lb.

$$\Sigma \vec{M_D} = -A_y(10) + 1000(6)$$

$\therefore A_y = 600$ lb

b/c we are analysising the entire truss as one structure, we can say

$\Sigma F_x = 0 = A_x \quad \therefore A_x = 0$

Now that we have known values we can look at each of the points.

Joint A,

$$\Sigma F_y = 0 = 600\vec{j} + T_{AB}\sin 56.3\vec{j}$$

$\therefore T_{AB} = -721$

$$\Sigma F_x = 0 = T_{AC}\vec{i} - 721\cos 56.3\vec{i}$$

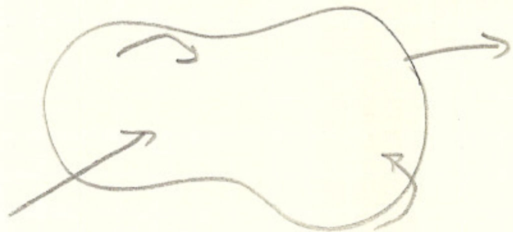
$\therefore T_{AC} = 400$

JOINT B.

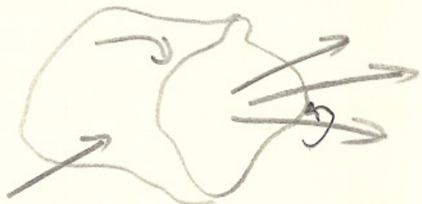
$$\Sigma F_x = 0 = -(-721)\cos 56.3^\circ + T_{BD}\cos\left(\frac{1}{5}\right)$$

.... we can go on and complete this problem.

CHAPTER 8, INTERNAL FORCES IN A STRUCTURAL MEMBER.



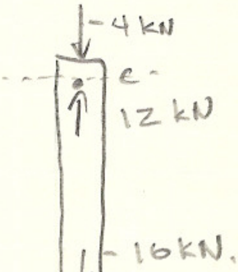
object ū forces (3D).



cut the object so we can see inside.

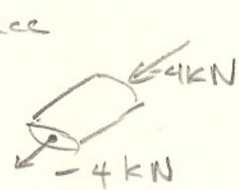
§ 8-2 AXIAL FORCE IN STRUCTURAL MEMBERS.

The force is passing through the centroid of the axial member.



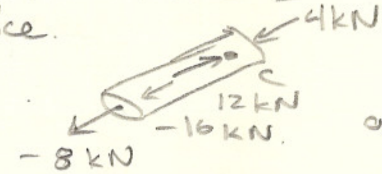
Determing the internal forces at B, C we take a cross section at pt C

top piece



way refers to compressive force.

cut of B, we look at upper peice.



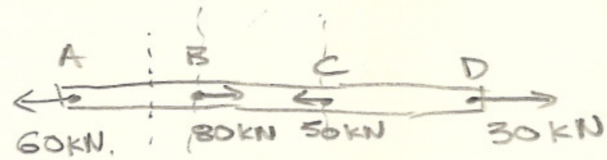
again we have compressive force

sign convention.

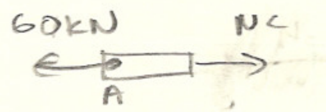
positive: tensile force

negative: compressive force

we use an axial force diagram

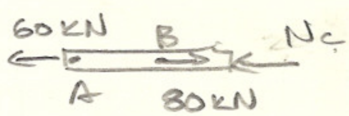


cross section of Between AB.



$\therefore\ 0 = -60\text{ kN} + N_c \qquad N_c = 60\text{ kN}.$

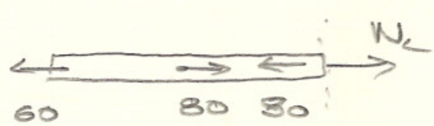
cross section of Between BC.



$\therefore\ 0 = -60 + 80 + N_c = \Sigma F$

$N_c = -20\text{ kN}.$

cross section of CD.



$\Sigma F = 0 = -60 + 80 - 50 + N_c$

$\therefore\ N_c = 30\text{ kN}.$

Assignment #4. (due friday July 21)

CH 6 - 32, 33, 35, 38, 44.

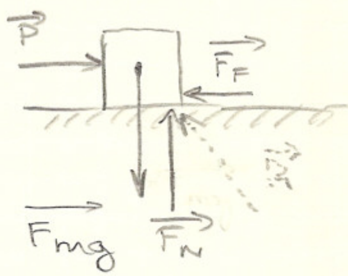
CH 7 - 2, 5, 7, 9.

CH 8 - 1, 3, 4.

CHAPTER 9 FRICTION.

§9.1 Characteristics of dry friction.

FRICTION: a force of resistance acting on a body which prevents or retards slipping of the body relative to a second body or surface which it is in contact.



if this diagram is in equilibrium then.

$\Sigma F_x = 0 \quad \therefore \vec{F_F} = -\vec{P}$

$\Sigma F_y = 0 \quad \therefore \vec{F_{mg}} = -\vec{F_N}$

- When $P=0$ then $F_F = 0$
- As force P i/cs the F_F i/cs.
- The Friction force has a maximum value F_{max}. The maximum value is called the limiting value of static friction.

- $F_{max} = \mu_s N$

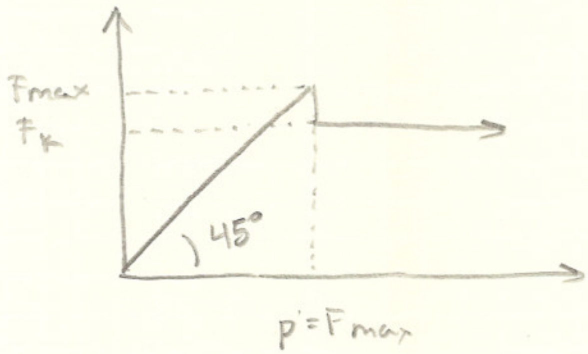
  μ_s: coefficient of static friction. The condition when the friction force is at it's maximum value, is called the condition of impending motion.

- If P i/c's beyond the point $p = F_{max}$, the block will start moving in the direction of force $\vec{P}$. The friction force of the contacting surface drops slightly to a smaller value F_K, called the kinetic friction force.

  $$F_K = \mu_K N$$

  $$(F_K < F_{max} = F_s)$$

  μ_K = the coefficient of kinetic friction.

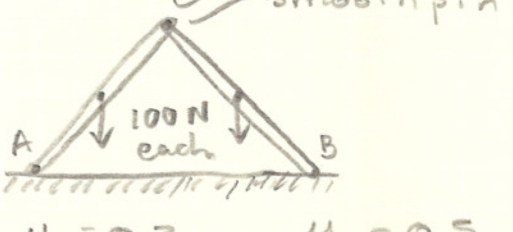


TYPE OF FRICTION PROBLEMS.

- number of unknowns.
- total number of equilibrium equations.

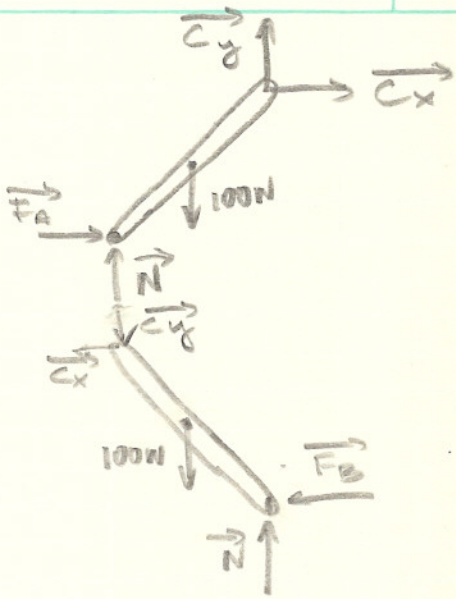
  - EQUILIBRIUM. (total number of unknowns is equal to the known.)

EX.



$\mu_A = 0.3$ $\mu_B = 0.5$

We want to determine the friction forces at A, B.

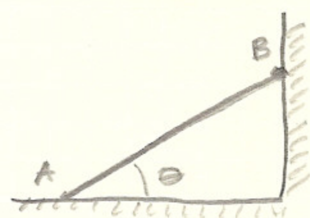


After solving friction force F. Their numerical value must be checked to be sure they satisfy the inequality $F \leq \mu_s N$. Otherwise slipping will occer.

- IMPENDING MOTION.

  The total number of unknowns will equal to the total number of equilibrium equations. plus the total number of availible friction equations.

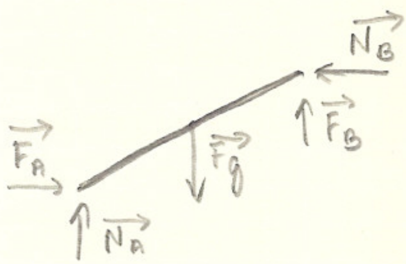
EX.



Find the smallest angle at which the ladder will not slip.

$\mu_A = 0.3$

$\mu_b = 0.4$



5 unknowns

3 equilibrium eqo.

$\Sigma F_x = \Sigma F_y = 0$

$\Sigma M_A = 0$

2 friction eqo.

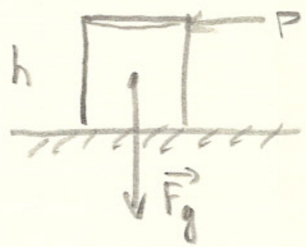
$F_A = \mu_A N_A$

$F_B = \mu_B N_B$

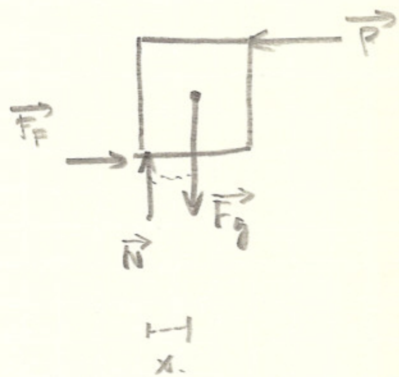
- Impending motion is known.

  total number of unknowns will be less than the total number of equilibrium equations plus the total number of friction equations or conditional equations for tipping

EX. Determine the force P needed to cause motion.



μ_s



4 unknowns

3 equilibrium equations.

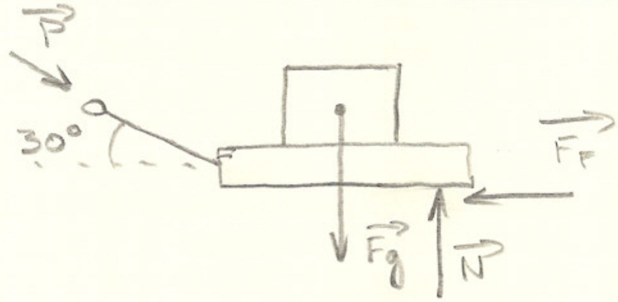
$+1 \begin{cases} \text{Slipping} & F_s = \mu_s N \\ \text{Tipping} & x = b/2 \end{cases}$

note that if you assume slipping, you must check that $x < \frac{b}{2}$

EX. A 20 lb block is placed on a wooden skid that weighs 10 lb that rests on a concrete floor. Assume $\mu_s = 0.45$ Determine.

a. the minimum pulling force nessasary to start the skid sliding.

b. the minimum force nessasary to start the skid in motion



$\vec{F_g} = 30$ lb.

3 uknowns

$\vec{F_F}, \vec{N}, \vec{P}$

A.

$\Sigma F_x = P\cos 30° - F_F$

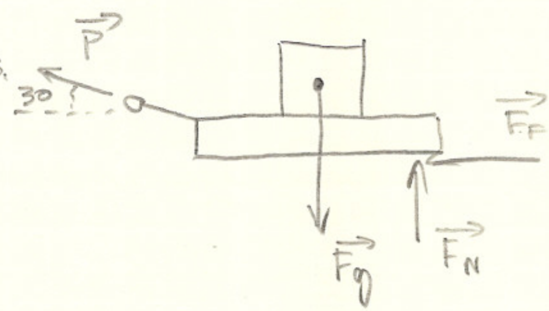
$F_F = \mu_s N$

$\therefore \Sigma F_x = P\cos 30° - 0.45N = 0$

$\Sigma F_y = N - 30 - P\sin 30° = 0$

$\therefore P = 21.1$ lb. $N = 40.5$ lb $F_F = 18.2$ lb

B.

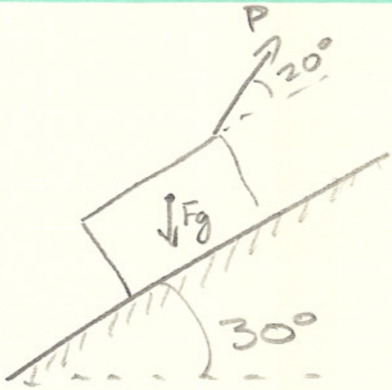


$\Sigma F_x = 0.45N - P\cos 30° = 0$

$\Sigma F_y = N + p\sin 30° - 30 = 0$

$\therefore P = 12.4$ lb $N = 23.8$ lb.

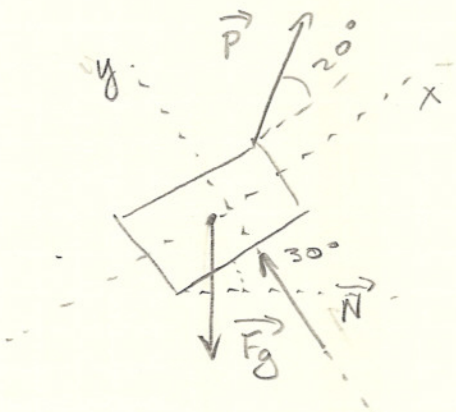
EX.



$\mu_s = 0.20$

$F_g = 100\,kg \cdot 9.81\,m/s$

$= 981\,N$



A.

$\Sigma F_x = 0 = \Sigma F_y$

assume, we will check later.

$\Sigma F_y = 0 = P \sin 20 - 981 \sin 30° + N$

$\Sigma F_x = 0 = P \cos 20° - 981 \cos 30° - F_F$

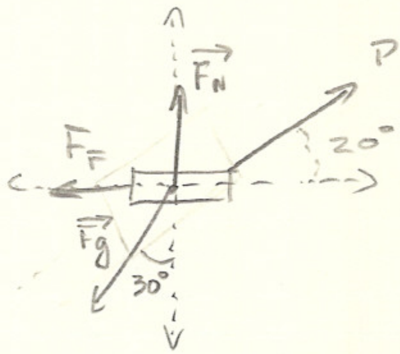
$\therefore P = 600\,N$

$N = 644\,N$

$F_F = 78$

$F_F < F_{MAX}$ it is in equilibrium

B.



$F_F = 0.2N.$

$\Sigma F_x = 0 = P\cos 20° + F_F - 981 \sin 30°$

$\Sigma F_y = 0 = P\sin 20° + N - 981 \cos 30°$

$P = 367.9N = \text{min.}$

C. $\Sigma F_x = 0 = P\cos 20° - F_F - 981 \sin 30°$

$\Sigma F_y = 0 = P \sin 20° + N - 981 \sin 30°$

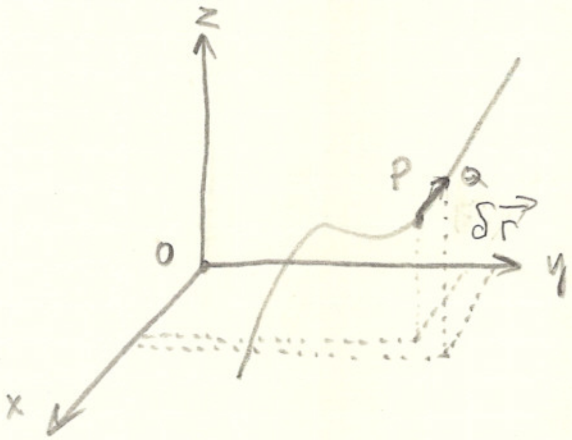
$P = 655.1\ N = \text{max}$

CHAPTER 13 KINEMATICS OF PARTICLES.

§13.1 position, velocity, acceleration.

A particle travels along a path. At some instant of time the particle is at P. In a fixed Cartesian coordinate system. The position of the particle is given by.

$$r_{P/O} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$



the displacement of a particle is the difference in position of the particle at two instants of time

t : at P

t + δt : at Q

displacement

$$\delta\vec{r} = \vec{r}_{Q/O} - \vec{r}_{P/O}$$

note: that no matter what the coordinate system the $\delta\vec{r}$ is the same.

The velocity of the particle is the time rate of change of position.

$$\vec{V}_P(t) = \lim_{\delta t \to 0} \frac{\delta\vec{r}}{\delta t} = \frac{d\vec{r}_{P/O}}{dt} = \dot{\vec{r}}_{P/O}$$

the acceleration of a particle is the time rate of change of velocity.

$$\vec{a_p}(t) = \frac{d\vec{V_p}}{dt} = \frac{d^2 \vec{r_{p/o}}}{dt^2} = \ddot{\vec{r}}_{p/o}$$

If $\vec{r_{p/o}} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

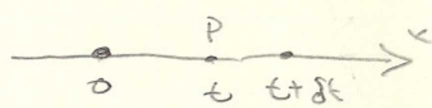
$$\dot{\vec{r}}_{p/o} = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j} + \dot{z}(t)\vec{k}$$

$$\ddot{\vec{r}}_{p/o} = \ddot{x}(t)\vec{i} + \ddot{y}(t)\vec{j} + \ddot{z}(t)\vec{k}$$

Types of motion we will discus.

- Rectilinear motion $y(t) = z(t) = 0$
- Curvelinear motion $z(t) = 0$.

§ 13.2 Rectilinear motion.



The distance travelled is different then the position. The distance travelled is equal to the total distace traveled.

$$d = \int_{t_1}^{t_2} |V(t)|\, dt.$$

- given $v(t)$

$$\dot{v}(t) = a(t)$$

$$\int v(t)\,dt = x$$

- given $a(t)$.

$$v(t) = \int a(t)\,dt$$

$$x(t) = \int\left(\int a(t)\,dt\right)dt$$

- given $a(x)$

$$a = \frac{d\,v(t)}{dt} = \frac{dv(t)}{dt}\cdot\frac{dx}{dx} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$\therefore\ a = \frac{v\,dv}{dx} \qquad a\,dx = v\,dv$$

$$\int a\,dx = \int v\,dv$$

$$\int a\,dx = \frac{1}{2}v^2$$

$$v = \sqrt{2\int a(x)\,dx}$$

- given $a = \text{const.}$

$$\frac{dv}{dt} = a$$

$$v = v_0 + a(t - t_0)$$

$$v - v_0 = a(t - t_0)$$

$$a = \frac{v - v_0}{t - t_0}$$

$$\frac{dx}{dt} = v = v_0 + a(t - t_0)$$

$$x = x_0 + v_0 t + \frac{a}{2}(t - t_0)$$

EX. A particle moves along y-axis with an acceleration given by $a(t) = 5 \sin \omega t$. (ft/s^2)

$\omega = 0.7$ rad/s

Initially $(t=0)$ the particle is at 2 ft above the origin and moving downward. with a speed of 5 ft/s

Determin

A. $v(t)$ & $x(t)$

B. displacement of the particle between $t=0$ & $t=4.5$.

C. the total distance travelled between $t=0$ & $t=4.5$

Solution.

A. $\frac{dv}{dt} = a$

$$dv = a\,dt$$

$$v = \int a\,dt$$

$$v - v_0 = \int_0^t 5 \sin \omega t \, dt$$

$$= -\frac{5}{\omega} \cos \omega t \Big|_0^t$$

$$v-(-5) = \frac{-5}{\omega}(\cos\omega t - 1)$$

$$v = \frac{-5}{\omega}(\cos\omega t - 1) - 5$$

or we can say.

$$\frac{dv}{dt} = 5\sin\omega t$$

$$v = -\frac{5}{\omega}(\cos\omega t) + C$$

$t=0 \quad v=5$

$$C = -5 + \frac{5}{\omega}$$

$$\therefore \quad v = \frac{-5}{\omega}\cos\omega t + \frac{5}{\omega} - 5$$

now finding. $y(t)$.

$$\frac{dy}{dt} = v = \frac{-5}{\omega}\cos\omega t + \frac{5}{\omega} - 5$$

$$y = \int\left(-\frac{5}{\omega}\cos\omega t + \frac{5}{\omega} - 5\right)dt$$

$$y = \frac{-5}{\omega^2}\sin\omega t + \frac{5t}{\omega} - 5t + C$$

$t=0 \quad y=2$

$$\therefore \quad y = \frac{-5}{\omega^2}\sin\omega t + \frac{5t}{\omega} - 5t + 2$$